Improved Maximum-Likelihood Time Delay Estimation for GPS Positioning in Multipath, Interference and Low SNR Environments

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Abstract—In this paper, we propose a robust maximum likelihood (RML) approach for delay and amplitude estimation of weak GPS signals in multipath and interference environments such as that encountered in indoor applications. First, to formulate the problem we exploit the fact that during the acquisition period the propagation delay causes only a circular shift to the spreading C/A code vector. This allows the decomposition of the received data vector into a constant component plus an undesired signals component. Then, a sample mean model is developed using a moving average over long time durations to reduce the effect of undesired zero-mean signals to one of a colored Gaussian noise. The developed model has an anti-jam structure which relies on the exploitation of a temporal structure property of the GPS signal, namely the replication of the C/A-code. An efficient temporal whitening technique is derived from the sample covariance matrix and applied to suppress the colored noise effect. The ML time-delay estimation of the superimposed multipath parameters becomes tractable and results in an efficient algorithm by adopting a sequential procedure.

I. INTRODUCTION

Despite the ever increasing civilian applications, significant limitations of the conventional GPS signal delay estimation arise from interference and, more importantly, multipath propagation as well as insufficient signal strength when operating indoor [7], [26], [29]. The effect of interference is to reduce the signal-to-noise ratio (SNR) of the GPS signal such that the GPS receiver is unable to obtain measurements from the GPS satellite [15], [26]. The spread-spectrum (SS) scheme, which underlines the GPS signal structure, provides a certain degree of protection against interference. However, when the interferer power is much stronger than the signal power, the spreading gain alone is insufficient to yield any meaningful information [26].

Interference can be combated in the time, space, or frequency domain, or in a domain of joint variables, e.g., time-frequency or space-time [15], [24]. It is typically mitigated prior to the correlation loops. In these loops, conventional GPS algorithms process blocks of one or several milliseconds of signal coherently and combine the results non-coherently [15]. For weak signals, receiver sensitivity can be amplified if longer durations of signal (e.g. 1 second or more) are processed coherently [22], [1], [6]. However, the existence of navigation bit transition limits the coherent integration period to 20 ms; normally 10 ms without the external aiding, using the current GPS L1 signal. In the case when the external aiding or assistance is not available, the navigation data can be estimated while the tracking process is on-going [31], [32]. The estimated binary data can be then used in a feedback scheme for the self-aiding to extend the coherent integration time of the tracking loop [31]. In this contribution, we assume knowledge of at least the end time of the current symbol, so the proposed RML acquisition algorithm can encompass the entire next symbol. This knowledge can be obtained using some sophisticated GPS techniques [6]. For indoor GPS applications, the data stream may be provided from cellular communication links [6], [7]. It is noted that the data stream is not present over a carrier in future GPS/Galileo signals [7]. For simplicity of presentation, the data modulation issue is not tackled in this paper, so as to demonstrate the potential of the proposed multipath mitigation method. Software approaches for the multipath problem have been proposed in recent years, including Narrow Correlator [27], Multipath Eliminating Technique (MET) [25], Multipath Eliminating Delay Lock Loop (MEDLL) [28], Pulse Aperture Correlator (PAC) [16], and other techniques that are based on the maximum likelihood theory [28], [23]. All existing methods are based on the Gaussian noise assumption and can’t cope with the cases when the GPS system receive other jammer and non-Gaussian interference signals.

In this paper, we propose a robust Maximum Likelihood (RML) time delay estimation for weak GPS signals in presence of multipath and interference. First, a stochastic mean model is derived and used to reduce the interference contribution to a colored noise effect. Then, the classical ML estimation problem of superimposed multipath signals in colored noise is efficiently solved by adopting a sequential procedure. The inverse of the estimated covariance matrix acts as a whitening filter that effectively suppresses the colored noise. The time delay of each path is estimated using an efficient recursive update. It should be noted that there is similarity in problem formulation to prior work in CDMA given in [4]. The contribution of this work is in the use of asymptotic Gaussian distribution of the averaged interference and noise term to simplify the estimation algorithm and the development of an alternative parametrization of the GPS signal component, which facilitates the use of computationally efficient iterative techniques for the complex ML optimization problem. This
idea is inspired from [4], in which the authors assume that the desired user, in the CDMA context, send a fixed preamble (i.e. a constant) during the acquisition. Here, we adopt the same approach for the GPS delay estimation problem by exploiting a large sample of data during which the C/A code sequence remains constant.

The GPS signal and A-GPS are described in Section II. In Section III, we develop an anti-jam mean model, and discuss a whitening mitigation of strong interferers. Section IV addresses the ML delay estimation. Computer simulations and the conclusions are presented in Section V and VI, respectively.

II. GPS System Model

A. Received GPS signal

In conventional GPS, the received signal, in multipath and interference environments, is an \( M \)-path model composed of the direct path signal and \((M-1)\) reflected rays, an interference term \( v(t)\), and the Gaussian additive noise \( \eta(t)\). In complex baseband, the signal is given by

\[
r(t) = \sum_{k=1}^{M} A_k e^{j\phi_k} s(t-\tau_k) + v(t) + \eta(t) \tag{1}
\]

In equation (1), \( s(t) \) is the signal of the given satellite, which is formed by spreading a binary phase-shift keying (BPSK) data stream \( \{b(i)\} \) onto a spreading waveform \( c(t) \) as

\[
s(t) = \sum_{i=-\infty}^{i=\infty} b(i)c(t-iT) \tag{2}
\]

where \( T \) is the C/A code period. If \( T_c = T/N \) the chip period, then

\[
c(t) = \sum_{n=0}^{N-1} c(n)\, rect(t-nT_c) \tag{3}
\]

where \( \{c(n)\} \in \{\pm 1\} \) is the pseudo-noise (PN) sequence of length \( N \) modulated with a rectangular chip-shaping pulse \( rect(\cdot) \). Each signal path wave is characterized by its relative amplitude \( A_k \), phase \( \phi_k \), and time delay \( \tau_k \), which are all assumed to be constant over the observation period. The interference term \( v(t) \) and the Gaussian noise \( \eta(t) \) are assumed to be zero-mean signals. Throughout this paper, we consider a chip period of \( T_c = 977.5 \) ns \((1023.10^6\) chips/second), which is the value used in commercial GPS. The GPS C/A code has a chip rate of \( 1/T_c = 1.023 \) MHz, one period of the spreading code consists of \( N = 1023 \) chips, hence it spans in 1 ms. Since each of the transmitted data bits in a GPS signal is repeated 20 times i.e. each bit is used to modulate 20 successive code periods, there are twenty 1 ms code epochs (blocks) in each GPS data bit.

B. Basics of Assisted GPS Algorithms

In A-GPS architecture, in addition to a GPS signal, several assistance data are made available to the localization algorithm: (i) Time stamp: represents an estimate of the time at which the GPS signal capture was initiated. In a CDMA network, time stamps are typically accurate within 100 \( \mu s \) or less. (ii) Location: typically taken to be the location of the base station from which the mobile device receives assistance data, the approximate location serves as a coarse estimate of the receiver, and (iii) Navigation data: that is required for coherent processing of long durations of signal [5], [6], [7], [22]. The job of an A-GPS is to estimate receiver location based on assistance data and the received GPS signal. It allows long coherent integration and increased SNR. Further, it avoids navigation data decoding to achieve acquisition [15].

Before applying the ML approach, we preprocess the observations to both overcome the boundary data bit problem and to apply a Gaussian model.

III. Weak GPS Signal Processing

A. Problem Formulation for Asynchronous GPS Receivers

In order to use long consecutive data blocks for improving SNR, a software approach will be undertaken in this contribution. We form a discrete-time observation by sampling the output of a filter matched to the chip waveform. In GPS acquisition mode, the chip timing of the user is unknown, and as such, the lower integration bound of the matched filter should take an arbitrarily value within a chip. In other words, the chip matched filter is not synchronized with any block interval (C/A-code period) or chip transitions. Accordingly, \( y_i(n) \), the \( n \)-th sample of the observation during the \( i \)-th block interval, is given by

\[
y_i(n) = \frac{1}{T_c} \int_{iT+nT_c}^{iT+(n+1)T_c} r(t) \, dt = x_i(n) + v_i(n) + \eta_i(n) \tag{4}
\]

where

\[
x_i(n) = \sum_{k=1}^{M} \frac{1}{T_c} \int_{iT+nT_c}^{iT+(n+1)T_c} \alpha_k c(t-iT-\tau_k) b(i) \, dt \tag{5}
\]

and \( \alpha_k \) is the \( k \)-th channel coefficient including amplitude and phase. We sample the received waveform at a rate \( Q/T_c \), where \( Q \) is an integer which represents the oversampling factor. Then, the received vector \( y_i \) is formed by stacking \( NQ \) samples from the oversampled discrete-time signal into a sequence of cyclostationary random vectors of length \( NQ \).

\[
y_i = [r(iNQ), r(1+iNQ), \cdots, r(NQ-1+iNQ)]^T \in \mathbb{C}^{NQ} \tag{6}
\]

where the \( n \)-th element is \( y_{i,n} = r(n+iNQ) \). In GPS acquisition mode, the chip timing of the user is unknown, and as such, the lower integration bound of the matched filter should take an arbitrarily value within a chip. To account for this fact, let us define the integer \( \nu_k \in \{1, \cdots, N\} \) and the fraction \( \gamma_k \in [0, 1) \) such that \( \tau_k = (\nu_k + \gamma_k)T_c/Q \). Then, we define \( \nu_k^{(v)} \) and \( \nu_k \) to be the augmented zero-padding sampled vectors of the C/A code waveform of the \( k \)-th GPS signal. They are both in \( \mathbb{R}^{2NQ} \), specifically they are defined for \( k = 0, \cdots, M \) as follow

\[
a_k^{(v)} \overset{\text{def}}{=} \begin{bmatrix} 0, & \cdots, & 0, & a(0), & \cdots, & a(N-1), & \cdots, & a(0), & \cdots, & 0 \end{bmatrix}^T \tag{7}
\]

\[
u_k \overset{\text{def}}{=} (1-\gamma_k) a_k^{(v)} + \gamma_k a_k^{(v+1)} \tag{8}
\]

Since the system is asynchronous, the observation vector will contain at least the end of the previous block and the beginning
of the current block. Thus, each vector can be viewed as a linear combination of two signal components, plus interference and noise. The power and phase may be collected into a single complex term $\alpha_k$ (channel coefficient). If the $i$-th block interval spans across two message data bits, $d(i-1)$ and $d(i)$, the vector $\mathbf{u}_k^i$ corresponds to the chip sequence of the previous data bit $d(i-1)$, and $\mathbf{u}_k^j$ corresponds to the chip sequence of the current data bit $d(i)$,

$$\mathbf{u}_k^i(\tau) = d(i-1)\mathbf{u}_k^i + d(i)\mathbf{u}_k^j$$

where the signal vectors

$$\mathbf{u}_k^i \overset{\text{def}}{=} [\mathbf{O}_{NQ} \mathbf{I}_{NQ}]\mathbf{u}_k^i \quad \text{and} \quad \mathbf{u}_k^j \overset{\text{def}}{=} [\mathbf{I}_{NQ} \mathbf{O}_{NQ}]\mathbf{u}_k^j$$

are the components of the $k$-th spreading waveform, where $\mathbf{O}_{NQ}$ denotes the $NQ \times NQ$ zero matrix and $\mathbf{I}_{NQ}$ denotes the $NQ \times NQ$ identity matrix. Accordingly, the $NQ \times 1$ sampled received vector $\mathbf{y}_i$ can be represented as

$$\mathbf{y}_i = \sum_{k=1}^{M} \alpha_k [d(i-1)\mathbf{u}_k^i + d(i)\mathbf{u}_k^j] + \mathbf{v}_i + \eta_i \quad (7)$$

We can write the above signal model (7) in the compact vector form,

$$\mathbf{y}_i = \mathbf{S}(\bar{\tau})\mathbf{a} + \bar{\mathbf{v}} + \bar{\eta} \quad (8)$$

where $\mathbf{S}(\bar{\tau})$ and $\mathbf{a}$ are defined by

$$\mathbf{S}(\bar{\tau}) = [\mathbf{u}_1^i \cdots \mathbf{u}_M^i \mathbf{u}_1^j \cdots \mathbf{u}_M^j]_{(NQ \times 2M)}, \quad \mathbf{a} = \begin{bmatrix} \bar{\alpha} \\ \mathbf{0} \end{bmatrix}_{(2M \times 1)}$$

$$\bar{\alpha} = [\alpha_1 \cdots \alpha_M]^T_{(M \times 1)} \quad \text{and} \quad \bar{d} = [d(i-1) \ d(i)]^T_{(2 \times 1)}.$$

The above model can be simplified by exploiting the C/A code repetition over the acquisition period. Consider the GPS signal component during initial acquisition period in which the satellite transmit a fixed-symbol C/A-code sequence of length $J$. Recall that in conventional GPS receiver $J$ is less than 20 ms, but one can use a longer duration as discussed in the introduction. That is, we shall assume that the data modulation is the all ones sequences (i.e. $d(i-1) = d(i) = 1$). Since the data bits and the channel parameters $\bar{\alpha}$ are constant during this period, the propagation delay $\tau$ causes only a circular shift to the spreading vector $c = [c(0), \cdots, c(N-1)]$. Therefore, the first term of the right side of (8) remains constant and this is the key idea behind the formulation of the proposed RML approach. To illustrate this fact, we define the circularly shifted spreading vector as

$$\bar{c}(\nu) \overset{\text{def}}{=} [c(0) \cdots c(N-1)]^T, \quad \text{if} \quad \nu = 0$$

$$\bar{c}(\nu) \overset{\text{def}}{=} [c(N-\nu) \cdots c(0) \cdots c(N-\nu-1)]^T, \quad \nu \geq 1$$

Then, we stack the circularly shifted versions of the spreading vector columnwise in a matrix $\mathbf{C} = [\bar{c}(0) \bar{c}(1) \cdots \bar{c}(N-1)]$. Equivalently, $\mathbf{C}$ is given by

$$\begin{bmatrix}
    c(0) & \cdots & c(N-\nu+1) & c(N-\nu) & \cdots & c(2) \\
    c(1) & \cdots & c(N-\nu+2) & c(N-\nu+1) & \cdots & c(3) \\
    \vdots & & \vdots & \ddots & \vdots & \ddots \\
    c(m) & \cdots & c(N-\nu+m) & c(N-\nu+m-1) & \cdots & c(m+1) \\
    c(N-1) & \cdots & c(N-\nu+m) & c(N-\nu+m-1) & \cdots & c(0)
\end{bmatrix}$$

The effective spreading code for the propagation delay $\tau$ is given by

$$\mathbf{C} \delta(\tau) = \mathbf{C} \begin{bmatrix} (1-\gamma) \\ \gamma \end{bmatrix} \quad \text{if} \quad \nu = \nu, \ (\nu+1)$$

$$= \mathbf{C} \begin{bmatrix} (1-\gamma) \mathbf{e}_\nu + \gamma \mathbf{e}_{\nu+1} \end{bmatrix} \quad (9)$$

where $\mathbf{e}_\nu$ is the $\nu$th unit basis vector in $\mathbb{R}^N$. Thus, the signal model can be obtained by summing the constant signal component $\mathbf{C} \Delta(\bar{\tau}) \bar{\alpha}$ and the contribution of noise $\eta_i$ and interferences $\mathbf{v}_i$ as

$$\mathbf{y}_i = \mathbf{C} \Delta(\bar{\tau}) \bar{\alpha} + \bar{\mathbf{v}} + \bar{\eta}_i \quad (10)$$

where $\Delta(\bar{\tau}) = [\delta(\tau_1), \cdots, \delta(\tau_M)] \in \mathbb{C}^{N \times M}$. The effect of the channel propagation (multipath and delay) to the spreading code remains constant during the acquisition period. This is a direct result from the fact that the effect of all multipath delays to the C/A code blocks was expressed in the constant matrix $\mathbf{C} \Delta(\bar{\tau})$. Actually, the first term in the above equation is a product of the constant C/A code constant matrix $\mathbf{C}$ and the channel effective filter $h(\bar{\tau}, \bar{\alpha}) = \Delta(\bar{\tau}) \bar{\alpha}$. Accordingly, an alternative representation for the received signal $\mathbf{y}_i$ is

$$\mathbf{y}_i = \mathbf{C} h(\bar{\tau}, \bar{\alpha}) + \bar{\mathbf{v}} + \bar{\eta}_i \quad (11)$$

Parametrization of $\mathbf{y}_i$ in terms of the unknown parameters $\bar{\tau}$ and $\bar{\alpha}$ can be easily achieved as

$$\mathbf{y}_i = \mathbf{s}(\bar{\tau}) \bar{\alpha} + \bar{\mathbf{v}} + \bar{\eta}_i \quad (12)$$

where $\mathbf{s}(\bar{\tau}) = \mathbf{C} \Delta(\bar{\tau})$. Next, we consider the problem of estimating the unknown parameters $\bar{\tau}$ and $\bar{\alpha}$ using the maximum likelihood approach and the signal model given in (12).

B. Stochastic Sample Mean Model

Due to the repetition of the spreading C/A code, the GPS signal exhibits strong self-coherence which can be seen as a "non zero-mean" property when averaging several blocks of data given by (12). This feature can be utilized to suppress a large class of zero-mean interferers. During the acquisition period, all interfering signals are assumed to be data modulated with zero mean. The interference can be narrowband or wideband signals, and can assume noise-like, chirp-like, pulse-like, or digital communication waveforms [24]. Since the GPS channel noise is also zero mean [15], from equation (12), the expected value of the observation vector $\mathbf{y}_i$ is

$$\mathbf{E}(\mathbf{y}_i) \overset{\text{def}}{=} \mathbf{m} = \mathbf{s}(\bar{\tau}) \bar{\alpha}.$$  

We can estimate $\mathbf{m}$ from the sample mean as,

$$\bar{\mathbf{m}}_J = \frac{1}{J} \sum_{j=1}^{J} \mathbf{y}_j = \mathbf{s}(\bar{\tau}) \bar{\alpha} + \mathbf{n}_J \quad (13)$$

where $\mathbf{n}_J \overset{\text{def}}{=} \frac{1}{J} \sum_{j=1}^{J} (\mathbf{v}_j + \eta_j)$. Clearly, we have

$$\mathbf{E}(\bar{\mathbf{m}}_J) = \mathbf{s}(\bar{\tau}) \bar{\alpha} = \mathbf{m}.$$
By the central limit theorem (CLT), \( \mathbf{n}_j \) converges in distribution to a complex Gaussian random vector (r.v.) as \( J \to \infty \). Therefore, \( \tilde{\mathbf{m}}_j \) is also asymptotically complex Gaussian r.v. with density function
\[
 f_{\tilde{\mathbf{m}}_j}(\tilde{\mathbf{m}}_j) = \lambda_j \exp \left\{ -[\tilde{\mathbf{m}}_j - s(\bar{\alpha})\bar{\alpha}^H \mathbf{K}_j^{-1}(\tilde{\mathbf{m}}_j - s(\bar{\alpha})\bar{\alpha})^\dagger \right\} \tag{14}
\]
where \( \lambda_j = \frac{1}{\det(\mathbf{K}_j)} \), \( \mathbf{K}_j \in \mathbb{C}^{NQ \times NQ} \) is the covariance matrix of rank \( K \), and \((.)^H\) represents the conjugate transpose operator. Because \( \mathbf{n}_j \) may contain correlated interference and noise, the assumption of white noise is not realistic. Thus, exploiting the particular "non-zero mean" nature of the GPS signal, we reduce the interference jammer effect to a colored Gaussian noise effect. This suggests the need for a robust whitening to decorrelate the data and improve the SNR.

C. Robust Whitening Preprocessing

The vector \( \mathbf{n}_j \) is contained in at most a \( K \)-dimensional subspace of \( \mathbb{C}^{NQ} \) with \( K < N \). The eigendecomposition of the covariance matrix is \( \mathbf{K}_j = \mathbf{V}_d \mathbf{D}_n \mathbf{V}_d^H \), where \( \mathbf{D} \) is a diagonal matrix of the eigenvalues \( \lambda_n \). Furthermore,
\[
\lambda_n = \begin{cases} \frac{1}{2} (d_n + \sigma^2), & \text{if } n \leq K \\ \frac{1}{2} \sigma^2, & \text{otherwise} \end{cases} \tag{15}
\]
where \( d_n \) is the variance of the interference along the \( n \)-th eigenvector and \( \sigma^2 \) is the Gaussian noise variance. Then, the \( K \) largest eigenvalues of \( \mathbf{K}_j \) correspond to the eigenvectors forming the subspace of interference signals spanned by \( \mathbf{V}_d \in \mathbb{C}^{NQ \times K} \). Thus \( \mathbf{V} \) can be partitioned as \( \mathbf{V} = [\mathbf{V}_d \mathbf{V}_N] \), where the columns of \( \mathbf{V}_N \) span the noise subspace. The matrix \( \mathbf{W} = \mathbf{D}^{-1/2} \mathbf{V} \) serves as a whitening filter for \( \tilde{\mathbf{m}}_j \). As the power of the interference term increases, the diagonal elements of \( \mathbf{D}^{-1/2} \) corresponding to \( \mathbf{V}_d \) approach zero. In the limit, the null space of \( \mathbf{W} \) converges to the interference subspace, i.e., the stronger the interference, the more it is suppressed by the whitening filter. The matrix \( \mathbf{K}_j \) can be estimated using the sample covariance matrix:
\[
\hat{\mathbf{K}}_j = \frac{1}{J} \sum_{j=1}^{J} \mathbf{y}_j \mathbf{y}_j^H - \tilde{\mathbf{m}}_j \tilde{\mathbf{m}}_j^H
\]
Compared to existing interference excision techniques [12], [2], [24], our procedure suppress the jammer without distortion nor energy reduction of the desired GPS signal.

IV. ML Time Delay Estimation

A. Single-Path Estimation

With a single-path channel (i.e. \( \bar{\tau} = \tau \) and \( \bar{\alpha} = \alpha \)), the ML estimation of \( \tau \) and \( \alpha \) based on the value of \( \tilde{\mathbf{m}}_j \) amounts to maximizing \( f_{\mathbf{m}_j, \tau, \alpha}(\tilde{\mathbf{m}}_j, \tau, \alpha) \) with respect to \( \tau \) and \( \alpha \). The solution of this standard signal parameter estimation in colored Gaussian noise is [19]
\[
\hat{\tau} = \arg \max_{\tau} \frac{|s(\tau)^H \mathbf{K}_j^{-1} \tilde{\mathbf{m}}_j|^2}{s(\tau)^H \mathbf{K}_j^{-1} s(\tau)} \tag{16}
\]
\[
\hat{\alpha} = \frac{s(\bar{\tau})^H \mathbf{K}_j^{-1} \tilde{\mathbf{m}}_j}{s(\bar{\tau})^H \mathbf{K}_j^{-1} s(\bar{\tau})} \tag{17}
\]
The conventional matched filter (MF) estimator is equivalent to assuming \( \mathbf{K}_j \) to be the identity matrix. Once the delay estimate \( \hat{\tau} \) is known, \( \hat{\alpha} \) can be immediately computed. Below, we propose a low-complexity implementation of the above equations (16) and (17). Next, we assume that the oversampling rate is \( Q = 1 \) for a simple presentation, otherwise one have to replace \( N \) by \( NQ \).

1) Time delay estimate computation: Ideally, we would like to differentiate the objective function (16) with respect to \( \tau \). However, the delay lies within an uncertainty region \( \tau \in [0, \bar{\tau}] \) and \( s(\tau) \) is only piecewise continuous on this interval. Furthermore, as illustrated in Fig. 1, this objective function generally has numerous local maxima. To counter these problems, we divide the uncertainty region into \( N \) cells of width \( T_c \) and consider a single cell \( C_\nu = [\nu T_c, (\nu + 1)T_c) \).

We again define \( \nu \in \{0, \ldots, N-1\} \) and \( \gamma \in [0, 1) \) such that \( \tau/T_c \mod (N) = \nu + \gamma \), and for \( \tau \in C_\nu \), the signal vector becomes
\[
s(\tau) = (1 - \gamma)s(\nu) + \gamma s(\nu + 1) \tag{18}
\]
and
\[
\frac{\partial s(\tau)}{\partial \tau} = s(\nu + 1) - s(\nu) = a \text{ constant}. \tag{19}
\]
Thus, within a given cell, we can differentiate the likelihood function and solve for the local maxima in closed form. We first compute the following 3\( N \) sufficient statistics:
\[
\begin{align*}
\hat{z}_1(\nu) &= s(\nu)^H \hat{\mathbf{K}}_j^{-1} s(\nu) \\
\hat{z}_2(\nu) &= s(\nu)^H \hat{\mathbf{K}}_j^{-1} s(\nu + 1) \\
\hat{z}_3(\nu) &= \hat{\mathbf{m}}_j^H \hat{\mathbf{K}}_j^{-1} \hat{\mathbf{m}}_j
\end{align*}
\]
Within the cell \( C_\nu \) we define
\[
\begin{align*}
a &= (z_1(\nu) - z_1(\nu + 1))\Re\{z_3(\nu)\bar{z}_3(\nu + 1)\} \\
&\quad + (|z_3(\nu + 1)|^2 - |z_3(\nu)|^2)\Re\{z_2(\nu)\} \\
&\quad - z_1(\nu)|z_3(\nu + 1)|^2 + z_1(\nu)|z_3(\nu)|^2 \\
b &= z_1(\nu)|z_3(\nu + 1)|^2 - z_1(\nu + 1)|z_3(\nu)|^2 \\
&\quad - 2\Re\{z_2(\nu)\} |z_3(\nu)|^2 \\
c &= z_1(\nu)\Re\{z_3(\nu)z_3(\nu + 1)\} - \Re\{z_2(\nu)\}|z_3(\nu)|^2
\end{align*}
\]
where \( \Re\{z\} \) denotes the real part of \( z \). The likelihood function has stationary point at \( \tau = (\nu + \gamma)T_c \), where
\[
\begin{cases}
a\gamma^2 + b\gamma + c = 0 \\
\gamma \in [0, 1)
\end{cases}
\]
Since \( C_\nu \) is a half-open interval, it need not contain a local maximum at all. However, if it does, that local maximum must occur at either \( \gamma = 0 \) or at one of the stationary points. Since the global maximum for the entire uncertainty region is by definition a local maximum of one of the cells, we have identified at most \( 3N \) candidates for \( \hat{\tau} \). We compute the likelihood function at each of these points and select the one with the greatest value.

The main computation burden lies in computing the sufficient statistics given in (20). However, low complexity computation can be implemented using recursive closed form calculations of equations (20). The key idea is using closed form expressions for the rank-one modification of a matrix
inverse based on the sample correlation matrix instead the sample covariance matrix, and exploiting the low complexity Cholesky factorization of the correlation matrix, which has a recursive expression

\[
\hat{R}_j = \frac{J - 1}{J} \hat{R}_{j-1} + \frac{1}{J} y_j y_j^H
\]

The details of this low complexity improvement are omitted and reported in a full-length journal paper version [20].

2) Covariance Estimation: Since for GPS applications, fast parameter estimation is desired, the sample covariance estimator has some limitations. Indeed, if the number of observation vectors is less than the code length \( N \), then the sample covariance matrix does not have a full rank. Thus, the ML cannot be computed until at least \( N \) blocks have been received. However, if \( K \ll N \), it should be possible to obtain a good estimate of the interference subspace from fewer than \( N \) vectors. Since only \( K \) vectors are needed to span \( \mathbf{V}_f \), the covariance matrix must have the form \( \mathbf{K} = \mathbf{B} \mathbf{B}^H + \sigma^2 \mathbf{I} \), where \( \mathbf{B} \in \mathbb{C}^{N \times K} \). Thus, we can estimate a matrix of this form using the least square error cost function,

\[
\begin{align*}
(\hat{\mathbf{B}}, \hat{\sigma}^2) &= \arg \min_{\mathbf{B}, \sigma^2} \| \hat{\mathbf{K}} - \mathbf{B} \mathbf{B}^H - \sigma^2 \mathbf{I} \|_2
\end{align*}
\]

Using the fact that the two-norm of a symmetric matrix is \( \max_n |\lambda_n| \), this problem has a simple solution

\[
\begin{align*}
\hat{\sigma}^2 &= \frac{1}{2} (\lambda_{K+1} + \lambda_N) \\
\hat{\mathbf{B}} &= [\mathbf{v}_1, \cdots, \mathbf{v}_K] \mathbf{D}_I^{1/2} 
\end{align*}
\]

where the \( \mathbf{v}_i \)'s are the eigenvectors of \( \mathbf{K} \) and \( \mathbf{D}_I = \text{diag}(\lambda_1 - \hat{\sigma}^2, \cdots, \lambda_K - \hat{\sigma}^2) \). Thus,

\[
\hat{\mathbf{K}} = \hat{\mathbf{B}} \hat{\mathbf{B}}^H + \hat{\sigma}^2 \mathbf{I}
\]

is equivalent to replacing the \( N - K \) smallest sample covariance eigenvalues by \( \hat{\sigma}^2 \). In using this covariance estimator, the interference subspace remains unchanged, so we have not compromised interference rejections. Yet, \( \hat{\mathbf{K}} \) has full rank and white noise subspace.

B. Multi-Path Estimation

1) ML for multipath parameters estimation: For a given pair \((\hat{\tau}, \hat{\alpha})\), the vector \( \hat{\mathbf{m}}_J \) is Gaussian with mean \( \mathbf{s}(\hat{\tau}) \hat{\alpha} \) and covariance \( \hat{\mathbf{K}}_J \). Thus the standard maximum likelihood estimator of \((\hat{\tau}, \hat{\alpha})\) is given by [11]

\[
\hat{\tau} = \arg \max_{\tau} \{ \Lambda(\tau) \}
\]

and

\[
\hat{\alpha} = [\mathbf{s}^H(\hat{\tau}) \hat{\mathbf{K}}^{-1} \mathbf{s}(\hat{\tau})]^{-1} \mathbf{s}^H(\hat{\tau}) \hat{\mathbf{K}}^{-1} \hat{\mathbf{m}}_J 
\]

where

\[
\Lambda(\tau) = \hat{\mathbf{m}}_J^H \Phi(\hat{\tau}) \hat{\mathbf{m}}_J 
\]

and \( \Phi(\hat{\tau}) \) is a square matrix defined by

\[
\Phi(\hat{\tau}) = \hat{\mathbf{K}}^{-1} \mathbf{s}(\hat{\tau}) [\mathbf{s}^H(\hat{\tau}) \hat{\mathbf{K}}^{-1} \mathbf{s}(\hat{\tau})]^{-1} \mathbf{s}^H(\hat{\tau}) \hat{\mathbf{K}}^{-1}
\]

Since there is no closed form solution to (25), the delays estimation requires a multidimensional search over the set spanned by \( \hat{\tau} = [\tau_1, \cdots, \tau_M]^T \). This makes the computational load prohibitive high even with few propagation paths. In the following, we propose a sequential procedure to reduce this multidimensional problem to a succession of \( M \) one-dimensional optimization problems. To estimate the delays, a simple grid search can be performed or one can apply the above recursive update method given by equations (20)-(22).

2) Sequential estimation of multipath parameters: To avoid the multi-dimensional search in (25) we introduce an iterative algorithm in which complex amplitude and delay of each path are estimated sequentially. The estimation procedure consists of \( M \) steps repeated several cycles as follow:

- **First cycle: parameter estimation.** At each step we look for the dominant path parameters.

  - Step 1: We estimate the first path parameters \((\hat{\alpha}_1, \hat{\tau}_1)\)

  - Step 2: We subtract the contribution of this path from the observation \( \hat{\mathbf{m}}_J \):

    \[
    \hat{\mathbf{m}}_J^{(2)} = \hat{\mathbf{m}}_J - \hat{\alpha}_1 \mathbf{s}(\hat{\tau}_1)
    \]

    Then we iterate the previous procedure over \( \hat{\mathbf{m}}_J^{(2)} \) instead of \( \hat{\mathbf{m}}_J \).

- Step \( m \) (\( 1 \leq m \leq M \)): Define

  \[
  \hat{\mathbf{m}}_J^{(m)} = \hat{\mathbf{m}}_J - \sum_{l=1}^{m-1} \hat{\alpha}_l \mathbf{s}(\hat{\tau}_l)
  \]

  and

  \[
  \hat{\tau}_m = \arg \max_{\tau} \frac{|\mathbf{s}(\tau)^H \hat{\mathbf{K}}_J^{-1} \hat{\mathbf{m}}_J^{(m)}|^2}{\mathbf{s}(\tau)^H \hat{\mathbf{K}}_J^{-1} \mathbf{s}(\tau)}
  \]

  \[
  \hat{\alpha}_m = \frac{\mathbf{s}(\hat{\tau}_m)^H \hat{\mathbf{K}}_J^{-1} \hat{\mathbf{m}}_J^{(m)}}{\mathbf{s}(\hat{\tau}_m)^H \hat{\mathbf{K}}_J^{-1} \mathbf{s}(\hat{\tau}_m)}
  \]

Step M: In the last step, we compute \((\hat{\alpha}_M, \hat{\tau}_M)\) to conclude the first cycle.

- **Second cycle:** refinement. We denote \((\hat{\alpha}_{m(1st)}^{(1st)}, \hat{\tau}_{m(1st)})\) to refer these estimates to the first cycle. In this 2nd cycle, we will refine the estimation procedure as follow. The main difference with the first cycle is that at each \( m \)-th step, all the paths with indices other than \( m \) are subtracted from \( \hat{\mathbf{m}}_J \) as

  \[
  \hat{\mathbf{m}}_J^{(m)} = \hat{\mathbf{m}}_J - \sum_{l \neq m} \hat{\alpha}_l \mathbf{s}(\hat{\tau}_l^{(1st)})
  \]

  and we compute the refined estimates \((\hat{\alpha}_m^{(2nd)}, \hat{\tau}_m^{(2nd)})\) using equations (27) and (28).

- **Last cycle of refinement.** Further cycles can be contemplated until reaching a convergence criterion. However, we remarked from simulations that two cycles are sufficient to achieve convergence.

V. SIMULATION RESULTS

We consider a data Gold code sequence composed of \( N = 1023 \) binary chips of time duration \( T_c = 977.5 \) ns; the block period is \( 1 \) ms. In the second and third experiments, we compare the proposed maximum likelihood algorithm RML with the conventional correlator receiver [15] and with the MF which corresponds to the particular case when \( \mathbf{K} = \mathbf{I} \), or
equivalently to assume that the interference plus noise component is white. The interference-to-signal ratio (ISR) is defined as $ISR = 10 \log_{10}(P_I/P_S)$, where $P_I$ is the interference power and $P_S$ is the signal power. Statistical performance has been obtained through 100 Monte Carlo runs.

- First experiment: Robustness to interference and colored noise effects. In this experiment, we demonstrate the receiver interference suppression capability using the sample mean model and the robust whitening technique. The interference assumes a sinusoidal signal [24]. As illustrated in Fig. 1 (the top), direct application of the log-likelihood function produces local maxima. Although it peaks at the correct delay $\tau = 0.4T_c$, there are numerous false peaks caused by the uneven weighting of the noise and interference subspace. Fig. 1 (the bottom), shows that the proposed preprocessing mitigates this problem and significantly reduces the false peaks. We have remarked also that when applying the conventional correlator, the delay estimation fails in presence of interference and/or non-Gaussian noise because it does not account for the non-Gaussian and high interference environment.

- Second experiment: Robustness to attenuation effects. In this experiment, the GPS signal encounters a variable attenuator, in order to reduce the signal power, before arriving to the GPS front end. The SNR of the received line-of-sight signal is -25 dB. For the no attenuation case, with 1 ms of data, the signal is easily identified and the time delay correctly estimated using the matched filter MF and the proposed RML. Although there is no attenuation in either algorithms, the resulting performance differs between the RML and the MF: the mean square error (MSE) of the MF estimate is 0.02 while it is equal 0.0010 for RML. This is a result of the statistical nature of each algorithm. After testing integration periods successively for the MF and the RML, the relatives necessary integration times for achieving a delay estimate with MSE less than 0.02 are given in the bellow table I.

Note that as the signal attenuation level is increased, the required integration time is also increased. This table shows that MF begins to fail when the signal is relatively poor (attenuated by 20 dB), whereas RML continues to produce good estimates using more integration time.

- Third experiment: Robustness to low SNR effect. The performance of both the proposed RML and MF estimators as a function of the SNR is illustrated in Fig. 2. One direct path scenario is considered, with $\tau = 0.25T_c$. The noise power is varied, according to SNR, and no interference is present. In moderate SNR, both RML and MF tend to have the same performance. However, as expected when the SNR becomes very small, the RML emerges as the superior solution thanks to the used mean model which cleans the averaged data before the maximum likelihood estimation task.

- Fourth experiment: Robustness to interference and multipath effects. In this experiment, we compare the time-delay estimation performance of the proposed robust ML (RML) scheme with the conventional ML [28] in a severe interference and multipath scenario. We consider a direct path and one reflective multipath such that the signal-to-multipath ratio (SMR) is 5 dB. The interference assumes a sinusoidal signal with ISR = 20 dB. In order to describe the behavior and to assess the performance of both the ML and the derived RML algorithm, we apply the root mean square error (RMSE). We average coherently over 100 code periods (i.e. $I = 100$ ms), which it is a reasonable observation interval [23]. Figure 3 depict the result of the estimate $\hat{\tau}$ in the case of one reflected path (i.e. $M = 2$). The conventional ML fails to estimate the time-delay if multipath signals arrives to the receiver with delays less than $0.9 T_c$ in presence of non-Gaussian interference. The RML estimator is biased for relative delays shorter than $0.5T_c$ as the ability of this algorithm to distinguish

<table>
<thead>
<tr>
<th>Attenuation level</th>
<th>Required time for MF</th>
<th>Required time for ML</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 dB</td>
<td>1 ms</td>
<td>1 ms</td>
</tr>
<tr>
<td>10 dB</td>
<td>2 ms</td>
<td>1.4 ms</td>
</tr>
<tr>
<td>15 dB</td>
<td>8 ms</td>
<td>6 ms</td>
</tr>
<tr>
<td>20 dB</td>
<td>-</td>
<td>18 ms</td>
</tr>
</tbody>
</table>

Fig. 1. Log-likelihood function calculated with covariance matrix estimated from 20 observation vectors. The parameters used are SNR = -20 dB, and ISR = 28 dB were set to match a range of values encountered in typical indoor GPS applications.

Fig. 2. MSE versus SNR.
the two paths is slowly vanishing towards shorter relative delays. The RMSE of the RML method for $\hat{\tau}_1$ is approximately between 5 and 20 meters, as the range of one chip for the GPS C/A code is 293 meters.

[Fig. 3. RMSE of the time-delay of the LOSS/Tc versus the relative time-delay of the multipath/Tc. Used parameters: $M = 2$, $J = 100$ ms, SNR = -20 dB, ISR = 20 dB and SMR = 5 dB.]

VI. ACKNOWLEDGEMENT

This work is supported by ONR/NWCCD under contract no. N65540-05-C-0028.

VII. CONCLUSION

A robust maximum likelihood approach for GPS delay estimation was proposed. It is based on applying a sample mean model to reduce all zero-mean interferences effect to a colored Gaussian noise effect. The used mean model relies on the repetitive structure of the C/A code which yields a non-zero outcome when averaging the code over its replicas. A whitening preprocessing was then used to mitigate the high colored noise associated with the interference. The latter may arise from operating indoor or in shadows of high-rising buildings. Another advantage of using a long time average (sample mean) lead to Gaussian random variables which, in turn, allow a ML solution. The weak GPS signal acquisition and tracking can be improved substantially with the aiding information provided by cellular networks, and/or exploiting the particular GPS temporal signal structure. The simulations results presented in this paper show promising results for an application in next generation of GPS/Galileo positioning at low SNR, despite power loss due to interference and building signal penetrations.

REFERENCES